



Research articles

Magnetic susceptibility of systems with different types of interactions: The random interaction fields method

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ABSTRACT

In the framework of the random interaction fields method, the behavior of magnetic susceptibility is described in ferromagnetic materials, antiferromagnetic materials, and spin glasses. Phase diagrams for systems with competing exchange interactions are built. The conditions for the occurrence of different types of magnetic ordering are defined. The ferromagnetic Curie points for different numbers of neighboring atoms are calculated.

1. Introduction

Spin glasses belong to the so-called atomic-disordered systems. The physics of such systems has actively been discussed lately [1–8]. Nowadays, many materials with spin-glass ordering are known. These materials include crystalline and amorphous substances, but most of them are diluted alloys. The occurrence of spin-glass ordering in these materials suggests that the reason for the absence of magnetic ordering is the decay of the percolation cluster into clusters of finite size. The short-range exchange interaction is absent in such clusters. The alternating sign of the Ruderman-Kittel-Kasuya-Yosida interaction or different signs of the exchange interaction for different pairs in amorphous alloys can be another reason.

It is natural to use methods based on the theory of random functions to describe disordered systems, because in these systems the local magnetization is a random function [1,2]. There are several models for describing spin glasses: the Edwards-Anderson model and the model of infinite radius proposed by Sherrington and Kirkpatrick [9]. There is also a heuristic model of the spin-glass state based on the results of numerical simulations [10].

The random interaction fields method enables us to simplify the description of spin glasses [11–21]. This method has the simplicity of the molecular field theory but at the same time enables us to determine the boundary of the transition from the state of magnetic ordering to the state of spin glass, at least for diluted magnets.

The main results of the random interaction fields method are as follows:

1. The distribution function has the form of a normal distribution, but unlike in the Edwards-Anderson model, the variance and the expectation are interrelated and are determined by the law of interaction. There is a critical concentration of interacting atoms. Magnetic ordering is absent when the concentration is lower than the critical concentration.
2. The method can be applied to systems in which the interaction between particles is described by different signs [22,23].
3. It enables us to investigate concentration magnetic phase transitions [23–25].
4. There is a possibility to account for variation in the Curie point by reason of the diffusion of magnetic particles [26].

In this article, we use the random interaction fields method to investigate magnetic susceptibility in ferromagnetic materials, antiferromagnetic materials, and spin glasses.

2. The random interaction fields method

2.1. Distribution density of random fields

We consider a system of exchange-interacting (“magnetic”) atoms

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that are randomly distributed with a concentration p in a nonmagnetic medium over the lattice sites. It is assumed that the magnetic moments of the atoms are oriented along a certain Oz axis (the Ising model approximation). Let us combine the origin with one of the magnetic atoms. Then, if we know the field $\varphi_k = \varphi_k(\mathbf{m}_k, \mathbf{r}_k)$, created at the origin by an atom with a magnetic moment \mathbf{m}_k and located at the point with coordinates \mathbf{r}_k , we can calculate the distribution density of random interaction fields $W(H)$ as

$$W(H) = \iint \delta\left(H - \sum_k \varphi_k(\mathbf{m}_k, \mathbf{r}_k)\right) \prod_k \Phi(\mathbf{m}_k, \mathbf{r}_k) d\mathbf{m}_k d\mathbf{r}_k, \quad (1)$$

where $\delta(H - \sum_k \varphi_k)$ is the distribution density of the field H with a known distribution of particles on the coordinates \mathbf{r}_k and magnetic moments \mathbf{m}_k , $\sum_k \varphi_k(\mathbf{m}_k, \mathbf{r}_k)$ is the total field from these particles, and $\Phi(\mathbf{m}_k, \mathbf{r}_k)$ is the density of the distribution of particles on the coordinates and magnetic moments.

If the magnetic moments of atoms do not depend on their coordinates, then

$$\Phi_k(\mathbf{m}_k, \mathbf{r}_k) = f(\mathbf{r}_k) \tau(\mathbf{m}_k). \quad (2)$$

Here, $f(\mathbf{r}_k)$ and $\tau(\mathbf{m}_k)$ are the density of the distribution of particles on the coordinates and magnetic moments, respectively. For crystalline ferromagnetic materials, the density of the distribution of particles on the coordinates has the form

$$f(\mathbf{r}_k) = \delta(\mathbf{r}_k - \mathbf{r}_{k,0}), \quad (3)$$

where $\mathbf{r}_{k,0}$ are lattice coordinates.

In the framework of the Ising model, the distribution density $\tau(\mathbf{m}_k)$ of atoms with the same magnetic moment $|\mathbf{m}_k| = m_0$ can be determined as follows:

$$\tau(\mathbf{m}_k) = [\alpha_k \delta(\theta_k) + \beta_k \delta(\theta_k - \pi)] [(1-p)\delta(m_k) + p\delta(m_k - m_0)], \quad (4)$$

where θ_k is the angle between \mathbf{m}_k and the axis Oz , α_k is the relative probability of the spin being "up" ($\theta_k = 0$), and $\beta_k = 1 - \alpha_k$ is the relative probability of the spin being "down" ($\theta_k = \pi$). In the field H , the equilibrium values of α_k and β_k are determined as follows:

$$\alpha_k = \frac{\exp\left(\frac{m_k H}{kT}\right)}{\exp\left(\frac{m_k H}{kT}\right) + \exp\left(-\frac{m_k H}{kT}\right)}, \quad \beta_k = \frac{\exp\left(-\frac{m_k H}{kT}\right)}{\exp\left(\frac{m_k H}{kT}\right) + \exp\left(-\frac{m_k H}{kT}\right)} \quad (5)$$

The problem of determining m is significantly simplified if we replace the values of α_k and β_k with their thermodynamically and configurationally averaged values in Eq. (5). Then, the magnetic moment has the form

$$M = \langle \alpha \rangle - \langle \beta \rangle = \int \tanh\left[\frac{m_0 H}{kT}\right] W(H, \langle \alpha \rangle, \langle \beta \rangle) dH. \quad (6)$$

2.2. Approximation of the "normal distribution" and the "rectangular" function

From Eqs. (1), (3), and (4) it follows that the characteristic function $A(\rho) = \int W(H) \exp\{i\rho H\} dH$ for a crystalline magnetic material can be represented as

$$A(\rho) = \prod_k [(1-p) + p \langle \alpha \rangle \exp\{i\rho \varphi_k(m_0, \mathbf{r}_{k,0})\} + p \langle \beta \rangle \exp\{-i\rho \varphi_k(m_0, \mathbf{r}_{k,0})\}], \quad (7)$$

$$\ln A(\rho) = \sum_k \ln[(1-p) + p \langle \alpha \rangle \exp\{i\rho \varphi_k\} + p \langle \beta \rangle \exp\{-i\rho \varphi_k\}]. \quad (8)$$

If we confine ourselves to the first three terms of the expansion of an exponential in a series, then we can obtain the following equation:

$$\ln A(\rho) \approx i \left[\langle \alpha \rangle - \langle \beta \rangle \right] p \sum_k \varphi_k \left[\rho - \frac{1}{2!} \left[1 - (\langle \alpha \rangle - \langle \beta \rangle)^2 p \right] p \sum_k \varphi_k^2 \right] \rho^2. \quad (9)$$

Hence, the distribution density of random interaction fields can be defined as

$$W(H) = \frac{1}{\sqrt{\pi B}} \exp\left\{-\frac{[H - H_0 M]^2}{B^2}\right\}, \quad (10)$$

where the values of H_0 and B^2 are determined via concentration p , φ_k , and $M = \langle \alpha \rangle - \langle \beta \rangle$, as follows:

$$H_0 = p \sum_k \varphi_k, \quad B^2 = 2p[1 - M^2 p] \sum_k \varphi_k^2. \quad (11)$$

This implies summation under the condition that all magnetic moments are directed upwards. Note that if we limited ourselves to the linear terms of the expansion, the distribution density would be a delta function of the form $\delta(H - MH_0)$. This function corresponds to the molecular field approximation.

From Eqs. (5) and (6), we can obtain the equation determining the dependence of the average magnetic moment on the temperature and concentration of the magnetic atoms. It has the form

$$m = \int \tanh\left[\frac{m_0 H}{kT}\right] W(H, M) dH, \quad (12)$$

where $W(H, m)$ is the density of distribution over the fields of interaction. This density can be determined from Eqs. 10,11.

A substantial simplification of Eq. (12) can be achieved by replacing the distribution function (10) with the rectangular function:

$$W(\tilde{H}) = \begin{cases} \frac{1}{2B}, & -B \leq \tilde{H} \leq B \\ 0, & \tilde{H} < -B, \quad \tilde{H} > B \end{cases} \quad (13)$$

Such a replacement can be justified only in the region of small m , that is, the region of phase transitions, where the error in the calculations becomes insignificant [11].

3. Magnetic ordering and phase diagram

If there are two sublattices, then in the external field h the corresponding equations can be written as follows:

$$M_1 = \frac{1}{2B_1} \int_{-B_1}^{B_1} \tanh\left[(H_1 + M_1 H_{011} + M_2 H_{012} + h) \frac{m_1}{kT}\right] dH_1. \quad (14)$$

$$M_2 = \frac{1}{2B_2} \int_{-B_2}^{B_2} \tanh\left[(H_2 + M_2 H_{022} + M_1 H_{021} + h) \frac{m_2}{kT}\right] dH_2. \quad (15)$$

For $M_1 H_{011} + M_2 H_{012} \ll 1$ and $h \ll 1$ from Eqs. (14) and (15) that

$$M_1 = \frac{1}{B_1} \tanh\left[\frac{m_1 B_1}{kT}\right] \left[H_{011} M_1 + H_{012} M_2 + h \right] - \frac{1}{3B_1} \left(\frac{m_1}{kT}\right)^2 \tanh\left[\frac{m_1 B_1}{kT}\right] (H_{011} M_1 + H_{012} M_2 + h)^3 + \frac{1}{3B_1} \left(\frac{m_1}{kT}\right)^2 \tanh^3\left[\frac{m_1 B_1}{kT}\right] (H_{011} M_1 + H_{012} M_2 + h)^3, \quad (16)$$

$$M_2 = \frac{1}{B_2} \tanh\left[\frac{m_2 B_2}{kT}\right] \left[H_{022} M_2 + H_{021} M_1 + h \right] - \frac{1}{3B_2} \left(\frac{m_2}{kT}\right)^2 \tanh\left[\frac{m_2 B_2}{kT}\right] (H_{022} M_2 + H_{021} M_1 + h)^3 + \frac{1}{3B_2} \left(\frac{m_2}{kT}\right)^2 \tanh^3\left[\frac{m_2 B_2}{kT}\right] (H_{022} M_2 + H_{021} M_1 + h)^3. \quad (17)$$

Let us consider these relations for $h = 0$. Defining in the zero

approximation the value of M_2 by the value of M_1 and the value of M_1 by the value of M_2 and substituting these in Eqs. (16) and (17), we obtain the equations for the average magnetic moments in the following form:

$$M_1^2 = \frac{3}{\left(\frac{m_1}{kT}\right)^2 \left[1 - \tanh^2 \left[\frac{m_1 B_1}{kT}\right]\right] \left\{ H_{011} + \frac{H_{012} H_{021} \tanh \left[\frac{m_2 B_2}{kT}\right]}{B_2 - H_{022} \tanh \left[\frac{m_2 B_2}{kT}\right]} \right\}^2} - \frac{3 B_1}{\left(\frac{m_1}{kT}\right)^2 \left[1 - \tanh^2 \left[\frac{m_1 B_1}{kT}\right]\right] \tanh^2 \left[\frac{m_1 B_1}{kT}\right] \left\{ H_{011} + \frac{H_{012} H_{021} \tanh \left[\frac{m_2 B_2}{kT}\right]}{B_2 - H_{022} \tanh \left[\frac{m_2 B_2}{kT}\right]} \right\}^3} \quad (18)$$

$$M_2^2 = \frac{3}{\left(\frac{m_2}{kT}\right)^2 \left[1 - \tanh^2 \left[\frac{m_2 B_2}{kT}\right]\right] \left\{ H_{022} + \frac{H_{012} H_{021} \tanh \left[\frac{m_1 B_1}{kT}\right]}{B_1 - H_{011} \tanh \left[\frac{m_1 B_1}{kT}\right]} \right\}^2} - \frac{3 B_2}{\left(\frac{m_2}{kT}\right)^2 \left[1 - \tanh^2 \left[\frac{m_2 B_2}{kT}\right]\right] \tanh^2 \left[\frac{m_2 B_2}{kT}\right] \left\{ H_{022} + \frac{H_{012} H_{021} \tanh \left[\frac{m_1 B_1}{kT}\right]}{B_1 - H_{011} \tanh \left[\frac{m_1 B_1}{kT}\right]} \right\}^3} \quad (19)$$

Because the difference in the right-hand side cannot be negative, we obtain the conditions for the occurrence of a nonzero average magnetic moment for both sublattices:

$$\frac{H_{011}}{B_1} \tanh \left[\frac{m_1 B_1}{kT}\right] \left\{ 1 + \frac{H_{012} H_{021} \tanh \left[\frac{m_2 B_2}{kT}\right]}{H_{011} B_2} \right\} \left\{ 1 - \frac{H_{022}}{B_2} \tanh \left[\frac{m_2 B_2}{kT}\right] \right\} > 1, \quad (20)$$

$$\frac{H_{022}}{B_2} \tanh \left[\frac{m_2 B_2}{kT}\right] \left\{ 1 + \frac{H_{012} H_{021} \tanh \left[\frac{m_1 B_1}{kT}\right]}{H_{022} B_1} \right\} \left\{ 1 - \frac{H_{011}}{B_1} \tanh \left[\frac{m_1 B_1}{kT}\right] \right\} > 1. \quad (21)$$

Based on the Eqs. (20) and (21), we can distinguish different cases of magnetic ordering. These can arise when the inequalities written below are fulfilled:

1. The absence of interaction between the sublattices ($H_{012} = H_{021} = 0$). Each of the sublattices has a ferromagnetic ordering when the temperature is lower than the Curie point:

$$\frac{H_{011}}{B_1} \tanh \left[\frac{m_1 B_1}{kT}\right] > 1, \quad \frac{H_{022}}{B_2} \tanh \left[\frac{m_2 B_2}{kT}\right] > 1. \quad (22)$$

2. The absence of interaction within the sublattices ($H_{011} = H_{022} = 0$). There is a negative exchange interaction between the sublattices. We obtain a ferromagnetic ordering when the temperature is lower than the critical temperature:

$$\frac{H_{012}}{B_1} \tanh \left[\frac{m_1 B_1}{kT}\right] \frac{H_{021}}{B_2} \tanh \left[\frac{m_2 B_2}{kT}\right] > 1. \quad (23)$$

3. The absence of interaction within the sublattices ($H_{011} = H_{022} = 0$ and $B_1 = B_2 = B$, $H_{012} = H_{021} = H$). There is a negative exchange interaction between the sublattices. We obtain an antiferromagnetic ordering:

$$\frac{H^2}{B^2} \tanh^2 \left[\frac{mB}{kT}\right] > 1. \quad (24)$$

Let us consider the case of ferromagnetic ordering. We set the concentration of magnetic atoms to $p = 1$. We also restrict our consideration to the vicinity of the critical point (near the phase transition). Then, taking into account Eq. (11), the moments of the distribution function can be determined as

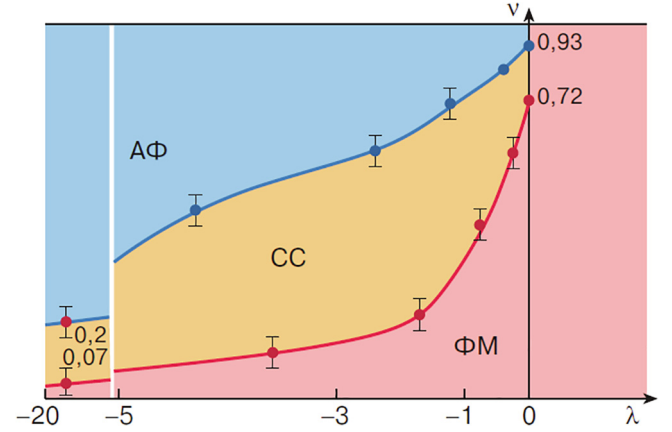


Fig. 1. The phase diagram obtained as a result of a numerical experiment for a magnetic state of a cubic magnetic material with stochastically mixed exchange bonds J and K ($\lambda = K/J$) on the plane (λ, ν) , where ν is the concentration of antiferromagnetic exchange bonds K [27].

$$H_0 = p \sum_k \varphi_k, \quad B = \sqrt{2p \sum_k \varphi_k^2}. \quad (25)$$

If we restrict ourselves to direct exchange ($\varphi_k = J$) for the neighboring atoms with coordination number z , the condition $\frac{H_0}{B} \tanh \left[\frac{B}{T}\right] > 1$ leads to the determination of the critical concentration $p_c = 2/z$. When the concentration is lower than the critical concentration, ordering is impossible. In this case, the percolation cluster is broken, and the spin glass appears. If there are competing interactions, we can use a model given in Ref. [27]. Fig. 1 shows the phase diagram obtained as a result of a numerical experiment for a magnetic state of a cubic magnetic material with stochastically mixed exchange bonds J and K ($\lambda = K/J$) on the plane (λ, ν) , where ν is the concentration of antiferromagnetic exchange bonds K [27].

There are three areas of magnetic ordering:

1. The ferromagnetic region is characterized by the predominant direction of elementary magnetic moments. In this case, the average value of the moment at the node is nonzero, and the system has a nonzero total magnetic moment in a finite temperature range.
2. The antiferromagnetic region corresponds to the ability to split the entire system into two sublattices. The average value of the moment at the node in each sublattice is also nonzero. However, the total magnetic moment of the entire system, in contrast to the ferromagnetic case, is zero as a result of the antiparallel orientation of the magnetic moments of these sublattices.
3. The spin glass region is characterized by the impossibility of separating the correct sublattices, that is, it exhibits complete stochastic spatial orientation of the magnetic moments of atoms. Obviously, the total magnetic moment of the magnet in the spin glass state is zero.

For amorphous magnetic materials, because of the different distances between the neighboring atoms, the interaction φ_k can be presented as:

$$\sum \varphi_k = K_1(1 - \nu)z - K_2 \nu z, \quad (26)$$

where K_1 is the positive exchange integral, ν is the concentration of exchange-interacting antiferromagnetic atoms, z is coordination number, and $(-K_2)$ is the negative exchange integral (introduced in such a way that $K_2 > 0$).

If we substitute the law of interaction (26) into the equations for the moments H_0 and B , we get

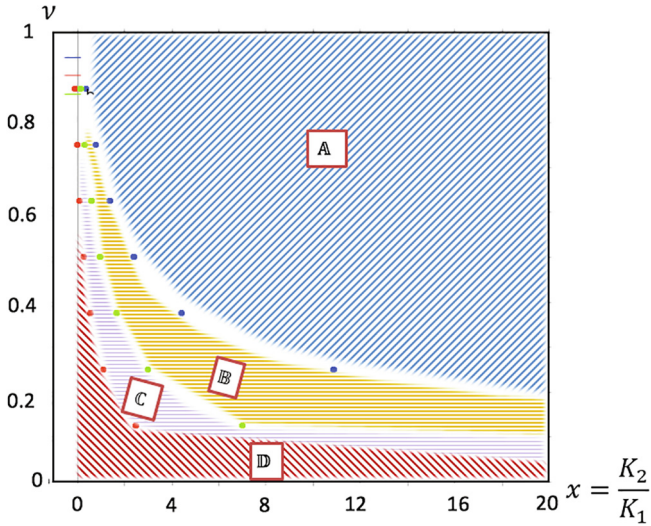


Fig. 2. Phase diagram for systems with competing exchange interaction $\nu(x)$, where A is an antiferromagnetic material, B is antiferromagnetic spin glass, C is a ferromagnetic spin glass, D is a ferromagnetic material.

$$\begin{aligned} H_0 &= K_1(1-\nu)z - K_2\nu z, \\ B &= \sqrt{2[K_1^2(1-\nu)z + K_2^2\nu z]}. \end{aligned} \quad (27)$$

Then, the ratio of the moments $\frac{H_0}{B}$, after simple mathematical transformations, has the form

$$\frac{H_0}{B} = \frac{(1-\nu)z - x\nu z}{\sqrt{2[(1-\nu)z + x^2\nu z]}}, \quad (28)$$

where $x = \frac{K_2}{K_1}$.

The ratio H_0/B allows us to differentiate the type of magnetic ordering as follows:

1. If $H_0/B > 1$, this is a ferromagnetic ordering.
2. If $H_0/B < 1$ and $H_0/B > 0$, this is a ferromagnetic spin glass state.
3. If $H_0/B < 0$ and $H_0/B > -1$, this is an antiferromagnetic spin glass state.
4. If $H_0/B < -1$, this is an antiferromagnetic ordering.

We construct our phase diagram (Fig. 2) for systems with a competing exchange interaction ν versus x using the ratio H_0/B (A is an antiferromagnetic ordering, B is an antiferromagnetic spin glass state, C is a ferromagnetic spin glass state, and D is a ferromagnetic ordering).

The results given in Ref. [27] are in good agreement with ours results, taking into account the sign of λ (in our case, $J = K_1$, $K = K_2$, $\lambda = x$).

The difference between Figs. 2 and 1 is as follows: In Fig. 2, the x axis corresponds to the positive value of the ratio $|K_2/K_1|$; therefore, the axis is directed to the right as compared with Fig. 1. In addition, Fig. 1 was obtained under the assumption of a random distribution of positive and negative bonds and our figure under the assumption of a random distribution of positively and negatively interacting nodes. Based on the theory of percolation, we know that a certain difference exists between these cases. Therefore, we believe that the correspondence between Figs. 2 and 1 is qualitative.

4. Magnetic susceptibility

Using the system of Eqs. (14, 15), we can determine the magnetic susceptibility of a ferromagnetic material in an external field h :

$$M = \frac{1}{2B} \int_{-B}^B \tanh \left[\frac{H + H_0 M + h}{kT} \right] dH. \quad (29)$$

Solving this equation for $M \ll 1$ and $h \ll 1$, we get

$$\begin{aligned} & \left[1 - \frac{H_0}{B} \tanh \frac{B}{T} \right] M \\ &= \frac{h}{B} \tanh \frac{B}{T} - \left[\frac{H_0^2 M^2}{BT^2} h - \frac{H_0^3 M^3}{3BT^2} \right] \left[\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right]. \end{aligned} \quad (30)$$

If the external field is equal to zero, then we have a formula for determining the magnetic moment in dimensionless variables ($k = 1$, $m = 1$):

$$M^2 = \frac{3 \left(\frac{H_0}{B} \tanh \frac{B}{T} - 1 \right)}{\frac{H_0^3}{BT^2} \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}. \quad (31)$$

Further, we solve the equation by the method of successive approximations $M = M_0 + \tilde{m}$. Considering only linear terms m with $h \ll 1$, we obtain the following equations:

$$\tilde{m} = \frac{\frac{h}{B} \tanh \frac{B}{T} - \frac{H_0^2 M_0^2}{BT^2} h \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}{1 - \frac{H_0}{B} \tanh \frac{B}{T} + \frac{H_0^3 M_0^2}{BT^2} \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}, \quad (32)$$

and

$$M_0^2 = \frac{3 \left(\frac{H_0}{B} \tanh \frac{B}{T} - 1 \right)}{\frac{H_0^3}{BT^2} \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}. \quad (33)$$

At $T < T_c$:

$$\tilde{m}_1 = \frac{\frac{h}{B} \tanh \frac{B}{T} - \frac{H_0^2 M_0^2}{BT^2} h \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}{1 - \frac{H_0}{B} \tanh \frac{B}{T} + \frac{H_0^3 M_0^2}{BT^2} \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}, \quad (34)$$

$$\chi_1 = \frac{\frac{1}{B} \tanh \frac{B}{T} - \frac{H_0^2 M_0^2}{BT^2} h \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}{1 - \frac{H_0}{B} \tanh \frac{B}{T} + \frac{H_0^3 M_0^2}{BT^2} \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}, \quad (35)$$

At $T > T_c$:

$$\tilde{m}_2 = \frac{\frac{h}{B} \tanh \frac{B}{T}}{1 - \frac{H_0}{B} \tanh \frac{B}{T}}, \quad (36)$$

$$\chi_2 = \frac{\frac{1}{B} \tanh \frac{B}{T}}{1 - \frac{H_0}{B} \tanh \frac{B}{T}}, \quad (37)$$

Formulas (35) and (37) make it possible to determine the dependence of the magnetic susceptibility χ on temperature T for various values of ν (Figs. 3 and 4).

Fig. 3 shows the dependence of the susceptibility χ on temperature

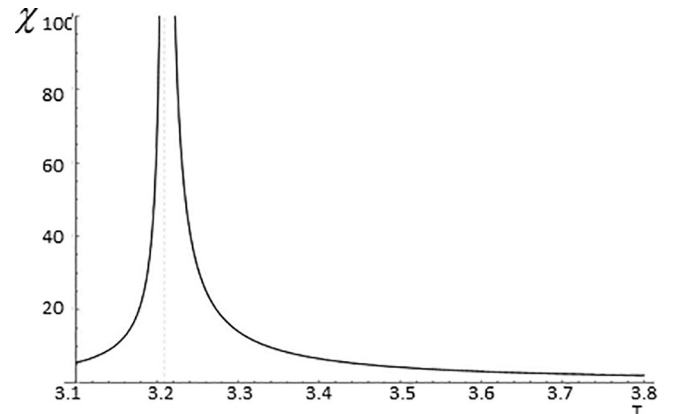


Fig. 3. Dependence of the magnetic susceptibility χ on temperature T ($\nu = 0$, $z = 4$, $T_c = 3.21$). Ferromagnetic region.

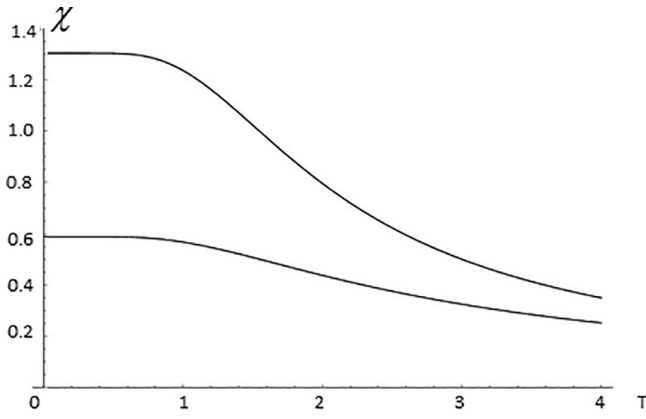


Fig. 4. Dependence of the magnetic susceptibility χ on temperature T ($\nu = 0.4$ and $\nu = 0.6$). Ferromagnetic spin glass region.

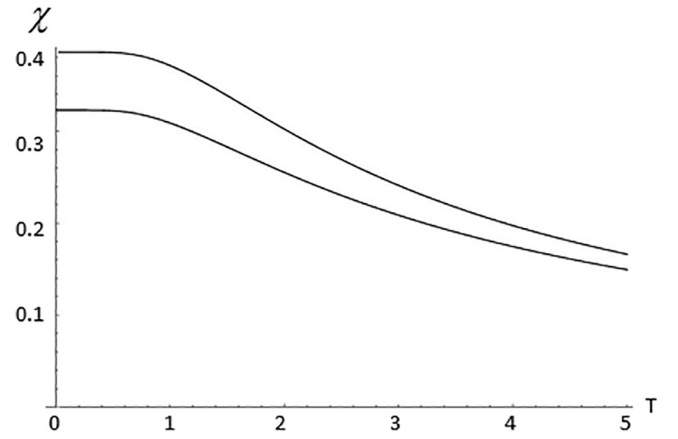


Fig. 5. Dependence of the magnetic susceptibility χ on temperature T ($\nu = 0.8$ and $\nu = 0.92$). Antiferromagnetic spin glass region.

T for a ferromagnetic ordering. For $\nu = 0$, $T_c = 3.21$; for $\nu = 0.23$, $T_c = 1.094$. As ν increases to 0.24, the Curie point shifts to the left. The curves χ_1 and χ_2 tend to infinity at the Curie point. For $T < T_c$, the susceptibility is determined by formula (35), but this formula is applicable only near the point T_c . For $T > T_c$, formula (37) is used. At high temperatures, the ferromagnet and ferromagnetic spin glass orderings become a paramagnetic ordering.

Fig. 4 shows the dependence of the susceptibility χ on temperature T at $\nu = 0.4$ and $\nu = 0.6$ for ferromagnetic spin glass. This follows from the condition that $H_0/B \tanh[B/T] < 1$. As can be seen from the figure, the susceptibility is finite.

Note that formula $\frac{1}{\chi_1} = \frac{1 - \frac{H_0}{B} \tanh \frac{B}{T}}{\frac{1}{B} \tanh \frac{B}{T}}$ at $B \rightarrow 0$ (i.e., the molecular field theory) tends to equation $\frac{1}{\chi_1} = T - H_0$. But the value of H_0 is the Curie point for the molecular field theory.

However, the Curie point T_c in the theory of random interaction fields corresponds to the condition $T_c < H_0$. The value of H_0 has the meaning of the ‘‘aramagnetic’’ Curie point T_{pm} , at which the short-range order is destroyed (for $T = T_c$, the long-range order is destroyed).

Both the ferromagnetic Curie point T_c and the paramagnetic temperature T_{pm} can be measured experimentally. The ferromagnetic Curie point can be determined from the equation $\frac{z}{\sqrt{2z}} \tanh \left[\frac{\sqrt{2z}}{T_c} \right] = 1$ at $p = 1$. By solving this equation numerically for T_c , we can determine the Curie ferromagnetic point T_c for different values of z . If we combine the random interaction fields method and the Bethe-Peierls method, it is necessary to use the additional factor $\frac{z-1}{z}$ when we calculate the Curie point [21].

We convert the obtained values of the ferromagnetic Curie point T_c into dimensional units. The results of calculating the Curie temperature T_c and the paramagnetic temperature T_{pm} , and the comparison of these temperatures with experimental data [28], are represented in Table (1), in which BP denotes the Bethe-Peierls method and MRF denotes the random interaction fields method.

Using the example of three substances Ni, Co, and Fe [28], the difference between the paramagnetic point and the Curie point is obtained experimentally (column 4), by means of the Bethe-Peierls method (column 5) and with our method (column 6). As shown in the table, the results we obtained are closer to the experimental

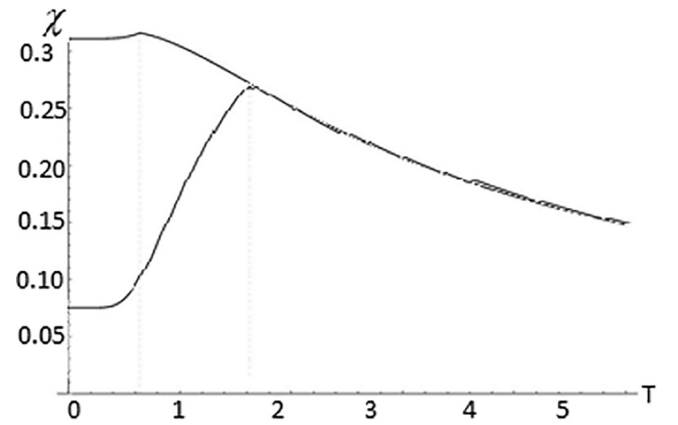


Fig. 6. Dependence of the magnetic susceptibility χ on temperature T ($\nu = 0.93$ at $T_N = 0.6405$ and $\nu = 1$ at $T_N = 1.605$). Antiferromagnetic region.

temperature difference compared with the Bethe-Peierls method. For comparison, the data are taken from Vonsovsky’s monograph since it refers to the fundamental manuals on magnetism.

The magnetic susceptibility in the region of the antiferromagnetic ordering can be determined if we consider the two-sublattice system 16–17 in the external field h . Then, considering only linear terms with $\tilde{m}_1 = \tilde{m}_2$ at $h \ll 1$, we get

$$\tilde{m}_1 + \tilde{m}_2 = 2 \frac{\frac{h}{B} \tanh \frac{B}{T} - \frac{J^2 M_0^2}{BT^2} h \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}{1 + \frac{J}{B} \tanh \frac{B}{T} - \frac{J^3 M_0^2}{BT^2} \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}, \quad (38)$$

$$\chi_1 = 2 \frac{\frac{1}{B} \tanh \frac{B}{T} - \frac{J^2 M_0^2}{BT^2} \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}{1 + \frac{J}{B} \tanh \frac{B}{T} - \frac{J^3 M_0^2}{BT^2} \left(\tanh \frac{B}{T} - \tanh^3 \frac{B}{T} \right)}, \quad (39)$$

where

Table 1
Paramagnetic and ferromagnetic Curie points.

| Substances | $T_{pm,ex}, ^\circ K$ | $T_{c,ex}, ^\circ K$ | $T_{pm,ex} - T_{c,ex}, ^\circ K$ | $T_{pm} - T_c, ^\circ K$ (BP) | $T_{pm} - T_c, ^\circ K$ (MRF) |
|-----------------|-----------------------|----------------------|----------------------------------|-------------------------------|--------------------------------|
| Ni ($z = 12$) | 650 | 631 | 19 | 108 | 38 |
| Co ($z = 12$) | 1428 | 1403 | 25 | 238 | 83 |
| Fe ($z = 8$) | 1101 | 1043 | 58 | 172 | 99 |

$$M_0^2 = \frac{3\left(\frac{J}{B}\tanh\frac{B}{T} - 1\right)}{\frac{J^3}{BT^2}\left(\tanh\frac{B}{T} - \tanh^3\frac{B}{T}\right)}. \quad (40)$$

In the absence of a spontaneous magnetic moment ($M_0 = 0$) at $T > T_N$, the magnetic susceptibility for an antiferromagnetic material will be determined as follows:

$$\chi_2 = \frac{2\frac{1}{B}\tanh\frac{B}{T}}{1 + \frac{J}{B}\tanh\frac{B}{T}}. \quad (41)$$

Fig. 5 shows the dependence of the susceptibility χ on temperature T at $\nu = 0.8$ and $\nu = 0.92$ for antiferromagnetic spin glass. This follows from the condition that $H_0/B\tanh[B/T] < 1$. $H_0 < 0$. As can be seen from the figure, the susceptibility is finite.

In the case of an antiferromagnet (Fig. 6), the curves χ_1 and χ_2 intersect at the Neel point, and the susceptibility is determined by formula (41).

At high temperatures, the antiferromagnet and antiferromagnetic spin glass states also become a paramagnetic state, as do the ferromagnet and ferromagnetic spin glass states.

The dependence of the magnetic susceptibility given in Figs. 3 and 6 is in good agreement with the theoretical concepts of the behavior of the corresponding susceptibility for the ferromagnet and antiferromagnet states [29,30]. If we consider spin glasses, we must consider that the formulas given in our work suggest that equilibrium has arrived and that magnetic susceptibility corresponds to the equilibrium state. However, in reality, this equilibrium can be achieved in a very long time. Thus, when trying to determine the magnetic susceptibility of spin glasses for a finite time, the following circumstances should be considered: Spin glass is a set of clusters oriented in our model partially up and partially down. In this sense, they are similar to a macroscopic antiferromagnet. And for a short research time, the magnetic susceptibility turns out to be similar in its behavior to the magnetic susceptibility of an antiferromagnet. Therefore, comparing our susceptibility with experimental data, this should be borne in mind. All experimental data are focused on a short research time. Therefore, in the area where we have an antiferromagnet, spin glass can show a magnetic susceptibility that is the same as an antiferromagnet in a short research time.

5. Conclusion

1. The random interaction fields method makes it possible to investigate the susceptibility of magnetic materials and separate the temperatures of occurrence of near and far orders. In addition, it enables us to determine the critical concentrations at which the percolation cluster is destroyed and ordering of the cluster spin-glass type occurs. If a ferromagnetic ordering exists, and when concentration p is lower than the critical concentration, the small clusters that were formed after the breakup of the percolation cluster retain the ferromagnetic ordering, which is destroyed at a higher temperature (ferromagnetic cluster spin glasses). If an antiferromagnetic ordering exists initially, then the small clusters, formed after the breakup, have antiferromagnetic ordering at a concentration p lower than the critical concentration (two equivalent sublattices with opposite spin directions, antiferromagnetic cluster spin glasses).
2. The general view of the dependence of the susceptibility on the temperature coincides qualitatively with the curve $\chi(T)$ given in Ref. [27]. In particular, there is a linear dependence $\chi(T)$ at low temperatures.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to

influence the work reported in this paper.

CRediT authorship contribution statement

V. Belokon: Conceptualization, Methodology, Software, Supervision. **R. Lapenkov:** Formal analysis, Writing - original draft, Writing - review & editing. **E. Chibiriak:** Investigation. **O. Dyachenko:** Software, Validation.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at <https://doi.org/10.1016/j.jmmm.2020.167051>.

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